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Minimum-Mass Truss Structures with Constraints on Fundamental Natural Frequency

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A technique for constrained parameter optimization is presented and applied to the minimum-mass design of truss structures. The procedure employs an exterior penalty function to transform the constrained objective function into an unconstrained index of performance which is minimized by the Gauss method. The Gauss method recasts the minimization problem to one of solving simultaneous linear equations with the variation of the parameters as the unknowns. The technique is first applied to several test problems, demonstrating its relative efficiency and accuracy. Next, the standard test problems are altered to introduce local buckling constraints and new designs are obtained. It is shown these designs also satisfy global stability. Finally, static thermal loads are introduced, and an equality constraint is imposed on the fundamental natural frequency of each structure. The natural frequency analysis uses a four-degree-of-freedom axial-force bar element.

Introduction

THE majority of publications concerning minimum-mass design have dealt with elastic truss structures subjected to static loads. Generally, the truss structure is modeled by axial-force bar elements. The objective function is the sum of the individual element masses. The design variables are the element areas, and inequality constraints are imposed on element stresses and nodal displacements. The resulting equations formulate a nonlinear mathematical programming problem.

Solutions of the nonlinear programming problem have been obtained by postulated criterion methods,[1-4] the generalized design variable method,[5] an optimality criteria method,[6] sequential linear programming methods,[7,8] and penalty function methods.[9]

In the penalty function method, a function of the inequality constraints are weighted and summed with the objective function formulating an unconstrained index of performance. Sequentially, the index of performance is minimized and the weight increased, until the constraints are satisfied to a desired accuracy.

Penalty function methods were among the first methods employed to seek minimum-mass designs. The attractions of the method include programming simplicity, while yielding a series of designs which converge to the optimum. However, the method presented a serious disadvantage. For a given design point, a step vector, representing the required relative changes in design variables, would first be calculated. Then a one-dimensional search for a minimum in the step direction would be performed. The one-dimensional searches required repeated calculation of the index of performance at different design points, rapidly accumulating a large number of structural analyses. This disadvantage led to the disfavor of the penalty function method, until Schmit and Miura incorporated approximate structural analysis with an interior penalty function approach.[9]

This paper investigates two areas. The first involves the use of a three-dimensional, four-degree-of-freedom axial-force bar element in natural frequency analysis. The four degrees-of-freedom consist of an axial displacement and strain (displacement derivative) at each end. Each degree-of-freedom has three components in the three global axis directions. The element may thus be considered as having 12 degrees-of-freedom. The second area involves the modification and application of a developed nonlinear programming procedure to the minimum-mass design of truss structures under static loads. Both of these areas are relevant to the minimum-mass design of large space structures, for which it is advantageous to accurately determine, or select, natural frequencies.

The motivation for using four-degree-of-freedom axial-force bar elements is to provide a substantial improvement in accuracy and efficiency in the natural frequency analysis of truss structures, as compared to that available when using the standard two-degree-of-freedom axial-force bar element.[10] This accuracy improvement is achieved while maintaining the relative ease of solution associated with an eigenvalue problem in which the stiffness and mass matrices are independent of natural frequency, as opposed to the use of the frequency dependent dynamic stiffness matrix presented by Liepins.[11]

The nonlinear programming procedure presented in this paper resulted from two observations. If the design variables are the square root of the element areas, then the minimum-mass design of truss structures seeks the minimum of a quadratic objective function subject to nonlinear constraints. If, in addition, an index of performance is formed by augmenting the objective function with a weighted sum of the squared-errors of the violated constraints, then the CONGAU algorithm of Ref. 12 may be employed to obtain a solution. The CONGAU algorithm demonstrates excellent performance in solving standard analytical test problems. With regard to obtaining the minimum-mass design of a truss structure, the algorithm has many beneficial features. Both inequality and equality constraints may be imposed on the objective function. The satisfied inequality constraints are not involved in the calculations to obtain a solution. An exterior penalty function is used, thereby allowing the solution attempt to begin with an infeasible design point. Using the square root of the element areas as design variables allows an element area to go to zero without any complications. The method requires only the first derivatives of the violated constraints. The readily available first and second derivatives of the objective function participate in the solution. This

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paper documents the modifications made to the basic algorithm of Ref. 12 to apply the procedure to minimum-mass design of truss structures. The method is applied to some standard problems to determine its efficiency relative to previously published solutions. The method is used to obtain new minimum-mass designs for the standard structures under local buckling constraints, combined thermal and mechanical loads, and with equality constraints on fundamental natural frequency.

Problem Statement

The minimum-mass design of homogeneous material truss-type structures subjected to a static load with constraints on displacement, stress, and fundamental natural frequency may be stated as follows. Minimize the objective function $f(D)$ where

$$f(D) = \sum_{i=1}^{n} \beta_i D_i^2, \quad D_i \geq D_{ci}, \quad i=1,\ldots,n \tag{1}$$

subject to

$$\begin{aligned} g_i &= 1-(\delta_i/d_i) \geq 0, \quad i=1,\ldots,p_d \\ g_i &= 1+(\delta_i/d_i) \geq 0, \quad i=p_d+1,\ldots,2p_d \\ g_i &= 1-(\sigma_i/c_i) \geq 0, \quad i=2p_d+1,\ldots,2p_d+p_s \\ g_i &= 1-(\sigma_i/t_i) \geq 0, \quad i=2p_d+p_s+1,\ldots,2p_d+2p_s \\ h_1 &= 1-(\omega^2/\omega_c^2) = 0 \end{aligned} \tag{2}$$

where

c_i = max $(-t_i, \; -\pi^2 E/s^2)$, maximum allowable compressive stress for ith element
d_i = maximum allowable displacement magnitude for ith degree of freedom
D_i = design variable, the square root of cross sectional area of a bar or the square root of a lumped mass
D_{ci} = lower limit on design variable
f = objective function, mass
L_i = length of ith element
p_d = number of constrained degrees of freedom
p_s = number of bars with stress constraints
R_i = radius of gyration of ith element
$s = L_i/R_i$, $i=1,\ldots,p_s$, selected design slenderness ratio
t_i = maximum allowable tensile stress for ith element
ω = fundamental natural frequency
ω_c = fundamental natural frequency constraint
β_i = mass linkage factor for ith design variable. If D_i^2 represents an element area, β_i is the sum of all the ρL_i's associated with D_i. If D_i^2 represents a lumped mass, β_i is the total number of nodes associated with D_i
δ_i = ith nodal displacement component
σ_i = axial stress in ith element
ρ = material mass density

Structural Analysis Procedure

The finite element model of a truss structure yields a system of equations relating the external force vector $\{F\}$, mass matrix $[M]$, stiffness matrix $[K]$, incremental stiffness matrix $[N]$, and the nodal displacement vector $\{\delta\}$

$$\{F\} = [[K]+[N]]\{\delta\} + [M]\{\ddot{\delta}\} \tag{3}$$

The general force vector may be written as

$$\{F\} = \{F_m\} + \{F_0\} \tag{4}$$

where $\{F_m\}$ represents the mechanical loads, and $\{F_0\}$ the internal loads. The internal loads result from initial strains. The contribution to nodal forces $\{F_0\}$ due to element k may be written as,

$$\{F_0\} = A_k \sigma_k^0 \begin{Bmatrix} \alpha \\ \lambda \\ \gamma \end{Bmatrix}_k \tag{5}$$

where A_k is the cross-sectional area; α, λ, γ represent the three direction cosines; and $\sigma_i^0 = E\epsilon_k^0$, with E being the modulus of elasticity and ϵ_k^0 the initial strain.

If the initial strain is the result of heating

$$\epsilon_k^0 = \alpha_T\{(T_i+T_j)/2 - T_r\} \tag{6}$$

where α_T is the coefficient of thermal expansion; T_i and T_j the nodal temperatures; and T_r the reference temperature.

Finally, the stress in element k spanning nodes i to j is given by a function of the nodal displacements u_i, u_j, v_i, v_j, w_i, and w_j, as

$$\sigma_k = \frac{E}{L_k} \lfloor \alpha \; \lambda \; \gamma \rfloor \begin{Bmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{Bmatrix}_k - \sigma_k^0 \tag{7}$$

Structural analysis problems reduce to obtaining analytical solution of forms of Eq. (2). The following algorithms outline some solution procedures:

Algorithm: Small displacement, static loads
- Step 1: Form $\{F\}$ and $[K]$
- Step 2: Solve $\{F\} = [K]\{\delta\}$
- Step 3: Solve for each element σ_k

Algorithm: Static global buckling load, $\{F_b\}$
- Step 1: Form $\{F\}$ and $[K]$
- Step 2: Solve $\{F\} = [K]\{\delta\}$
- Step 3: Solve for each element σ_k
- Step 4: Form $[N]$ with $\sigma_k^0 = E\epsilon_k^0$
- Step 5: Solve $[[K]+\phi[N]]\{\delta\} = \{0\}$, $\{F_b\} = \phi\{F\}$

Algorithm: Natural frequency ω
- Step 1: Form $[K]$ and $[M]$
- Step 2: Solve $[[K]-\omega^2[M]]\{\delta\} = \{0\}$

The solution for the static global buckling load assumes no individual element fails due to local buckling.

Martin[13] presented a derivation of stiffness matrices necessary for the analysis of two-dimensional deflection and stability problems associated with truss structures. Yang and Sun developed a one-dimensional four-degree-of-freedom axial-force bar element that demonstrated excellent performance in the axial free vibration analysis of a cantilever bar.[10] The four degrees-of-freedom consist of an axial displacement and an axial strain (displacement derivative) at each of the two nodal points. The present paper makes use of Martin's derivation, and the four degrees-of-freedom element, extended to three dimensions. Such a bar element then has six degrees-of-freedom at each end: three orthogonal displacement components and three orthogonal strain components, all with reference to global coordinates.

When using such 12-degree-of-freedom bar elements to model truss structures, there appear to be two common options in treating the strain (displacement derivative) degrees-of-freedom. The three orthogonal strain components at each common nodal point for all the adjoining elements can be assumed as compatible. On the other hand, they can also be assumed as incompatible at the common nodal point. This can be done by simply writing the strain degrees-of-freedom in terms of the displacement degrees-of-freedom through matrix reduction and thus eliminating them at the element level before assemblage. The former method was used in performing the structural analyses for the present examples.

Optimization Method

For simplicity of presentation, brackets, semi-brackets, and braces that are used in the previous section to symbolize matrix, row vector, and column vector, respectively, are omitted in this section.

The CONGAU[12] algorithm seeks the minimum of an objective function

$$f(D) = \Theta^t(D)\Theta(D) \tag{8}$$

where $\Theta^t(D) = (\Theta_1(D),...,\Theta_m(D))$; and $D^t = (D_1,...,D_n)$ subject to

$$h(D) = \begin{Bmatrix} h_1(D) \\ h_2(D) \\ \vdots \\ h_p(D) \end{Bmatrix} = 0 \tag{9}$$

$$g(D) = \begin{Bmatrix} g_1(D) \\ g_2(D) \\ \vdots \\ g_q(D) \end{Bmatrix} \geq 0 \tag{10}$$

The solution is obtained by finding minima of an index of performance P, where

$$P(D,w) = \Theta^t(D)\Theta(D) + w[h^t(D)h(D) + \ell^t(D)\ell(D)] \tag{11}$$

The index of performance represents a standard exterior penalty function form consisting of the scalar penalty weight w and the squared-error of the unsatisfied constraints e, where

$$e = h^t h + \ell^t \ell \tag{12}$$

The column matrix ℓ consists of the elements of g which violate the inequality constraint.

The following describes the basic CONGAU algorithm:

Step 1: Select starting point D, initial weight w, and tolerances γ_1 and γ_2.

Step 2: Calculate matrix B and column vector b, where

$$B = J^t J + w(U^t U + V^t V) \tag{13}$$

$$b = -[J^t\Theta + w(U^t h + V^t \ell)] \tag{14}$$

and

$$J = \partial\Theta/\partial D \tag{15}$$

$$U = \partial h/\partial D \tag{16}$$

$$V = \partial g/\partial D \tag{17}$$

Step 3: Solve $B\delta D = b$ for δD. Calculate $D' = D + \delta D$.

Step 4: Sequence termination check. If $\{P(D') - P(D)\}/P(D) > \gamma_1$, then set $D = D'$ and repeat from step 2. Otherwise, go to step 5.

Step 5: Termination check. If $e(D') < \gamma_2$, terminate with D' as the solution. Otherwise, set $w = cw$ $(c > 1)$, set $D = D'$, repeat from step 2.

With the design variable D representing the square root of element areas, or the square root of the lumped masses, the form of Eq. (1) allows for the implementation of the CONGAU algorithm. It should be noted that $\nabla f^t = (2\beta_1 D_1,...,2\beta_n D_n)$, and $J^t J$ is used as an approximation for the Hessian of f, $\nabla^2 f = \text{diag}\,(2\beta_1,..., 2\beta_n)$. Since the Hessian of f is readily available, it is used in place of $J^t J$.

The first derivatives are obtained analytically from the following equations

$$K\frac{\partial\delta}{\partial D_i} = \frac{\partial F}{\partial D_i} - \frac{\partial K}{\partial D_i}\delta \tag{18}$$

$$\frac{\partial\sigma}{\partial D_i} = \frac{E}{L}\lfloor \alpha \;\lambda\; \gamma \rfloor \frac{\partial}{\partial D_i}\begin{Bmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{Bmatrix} \tag{19}$$

$$\frac{\partial\omega^2}{\partial D_i} = Y^t\left(\frac{\partial K}{\partial D_i} - \omega^2\frac{\partial M}{\partial D_i}\right)Y, \quad Y^t M Y = 1 \tag{20}$$

The basic CONGAU algorithm was modified to improve performance when considering specific features of the minimum-mass design problem. These modifications concern: 1) scaling the objective function; 2) selection of the initial penalty weight; 3) introduction of one-dimensional minimization procedures; 4) design variable constraints; and 5) penalty weight modification.

The objective function is scaled by setting $\beta_i = \beta_i/f'$, where f' is the initial value of f. The minimization procedure is not initiated until $0 < e < 2$. If necessary, this is accomplished by the repeated application of one of the following means. If only stress constraints are active, the area design variables are altered in accordance with a fully stressed design iteration. If any nonstress constraint is active, the area design variables are doubled. The initial penalty weight is chosen so that $w = \max(10, 1/e)$. Two one-dimensional search procedures are introduced. The first is used if an iteration results in an index of performance value exceeding the previous iteration value. If so, the values of the two indices of performances and local gradient information are used to construct a one-dimensional quadratic approximation of the index of performance. From this approximation, the local minimum is determined and the next iteration proceeds from this design point. The second one-dimensional search procedure is introduced with an approximate structural analysis routine to obtain the minimum in the current step direction. This one-dimensional minimization is carried out using a golden section search.[14] The design variable constraints are accommodated solely by restricting the magnitude of the variations so no constraint is ever violated. The penalty weight is modified so that $w = \max(10w, 1/e)$.

In formulating the solution procedure, computational efficiency was gained by:

1) Storing for each element only the 6 distinct components of the 36 component two-degree-of-freedom element stiffness matrix, and storing only 42 of the 144 components of the four-degree-of-freedom stiffness matrix and mass matrix.

2) Applying the boundary conditions during the formulation of system matrices, force vector, and the vectors: $(\partial K/\partial D_i)\delta$, $(\partial\delta/\partial D_i)$, $(\partial K/\partial D_i)Y$, $(\partial M/\partial D_i)Y$, and $(\partial\sigma/\partial D_i)$.

3) Working with the upper triangle of the symmetric stiffness and mass matrices, stored as vectors.

4) Calculating the vectors: $(\partial K/\partial D_i)\delta$, $(\partial K/\partial D_i)Y$, $(\partial M/\partial D_i)Y$, avoiding the storage of the sparse matrices $(\partial K/\partial D_i)$ and $(\partial M/\partial D_i)$.

5) Employing a simultaneous linear equation solver with provision for multiple constant vectors to obtain $(\partial\delta/\partial D_i)$ from Eq. (18), by back substitution once $\{F\} = [K]\{\delta\}$ is solved.

Table 1 Minimum-mass design of three-bar truss

	Standard design, $s=0$[a]			OPTRUS design	
	Exact	Ref. 3	Ref. 4	$s=0$	$s=150$
Areas A_1 and A_3, cm^2	5.08853	5.058	5.142	5.09176	5.42783
Area A_2, cm^2	2.63402	2.723	2.684	2.62438	2.24468
Mass, kg	11.970	11.973	12.112	11.970	12.374
Structural analyses		99	13	6	9
Optimization analyses		0	13	4	6
Final squared error	0	0	0	7.6×10^{-10}	4.9×10^{-9}

[a] Slenderness ratio.

Table 2 Minimum-mass design of four-bar truss

	Standard design, $s=0$[a]		OPTRUS design	
	Ref. 1	Ref. 8	$s=0$	$s=150$
Areas, cm^2				
A_1	20.304	20.408	20.4581	17.1857
A_2	17.362	17.227	17.1599	84.4922
A_3	13.956	13.930	13.9296	79.4022
A_4	0.006	0.0	0.0	0.0
Mass, kg	58.328	58.279	58.269	204.264
Structural analyses	8	8	9	12
Optimization analyses	8	8	6	8
Final squared error	0	7.9×10^{-8}	1.5×10^{-7}	2.6×10^{-13}

[a] Slenderness ratio.

Numerical Examples

The above procedure was formulated as two FORTRAN IV programs entitled OPTRUS and OPTRUS2. The application of the OPTRUS program is limited to truss structures under a static mechanical load, with design variable, displacement, and stress constraints. The results of the application of the OPTRUS routine allow for the comparison of the method with previously published work. The OPTRUS routine is also used to obtain designs of these structures while including local buckling constraints. It is shown that these particular structures are globally stable. The OPTRUS2 program introduces solutions of Eq. (1) for these structures under a mechanical and thermal load, with an equality constraint on the fundamental natural frequency. Both programs are formulated to take advantage of the discussed computational efficiency procedures.

Both programs solve the necessary simultaneous equations based on a symmetric decomposition algorithm presented by Melosh and Bamford.[15] The fundamental natural frequency and corresponding eigenvector were obtained by the power method of Mises.[16]

Global buckling information was obtained from a third program, entitled TRUSAN. TRUSAN consists of the structural analysis portion of OPTRUS2, with the additional capability of solving the complete eigenvalue problems associated with static stability and free vibration. These eigenvalue problems are solved by using the generalized Jacobi method[16] on the upper triangle of the symmetric matrices, stored as vectors. For all the structures, aluminum is the design material, with $E=68{,}966$ MPa, $\rho=2767.9$ kg/m^3, and $\alpha_T=22.5\times10^{-6}$/K.

The Purdue University Computing Center CDC 6500 was employed in all executions of OPTRUS, using single precision arithmetic and compiled with the Minnesota Fortran Compiler. The Rockwell International Corporate Computing Center CDC Cyber 176 was employed in all executions of TRUSAN and OPTRUS2, using single precision arithmetic and the CDC supplied FTN compiler.

OPTRUS Applications

The standard design problem considers only the mechanical load with compressive and tensile stress constraints of equal absolute value. In the formulation of Eq. (1), this is equivalent to setting the selected slenderness ratio to zero. Imposing $s=150$ restricts the magnitude of the compressive stress to be less than 19,519.88 N. The specified slenderness ratio also sets the minimum acceptable radius of gyration for each element. With a specified area, the minimum radius of gyration sets the minimum area moment of inertia for an element cross section. It is possible no cross section will simultaneously meet the area and minimum radius of gyration requirements. Then, the design material or the slenderness ratio requirement must be altered to meet the local buckling constraint.

Three-Bar Truss

The structural geometry, constraints, and mechanical loads are as defined in Ref. 3. The minimization procedures initiated with $A_1=A_3=12.904$ cm^2 and $A_2=6.452$ cm^2. A comparison of minimum-mass designs is presented in Table 1. The squared-error for the OPTRUS standard design resulted from the horizontal displacement, and the tensile stress in bar 1. The squared-error for the OPTRUS local buckling constrained design resulted from the vertical displacement, and tensile stress in bar 2. OPTRUS required 0.345 CPU seconds for the solution of the $s=0$ problem, and 0.377 CPU seconds for the solution of the $s=150$ problem.

Four-Bar Truss

The structural geometry, constraints, and mechanical loads are as defined in Ref. 1. The minimization procedures initiated with all $A_i=17.206$ cm^2. A comparison of minimum-mass designs is presented in Table 2. The squared-error for the OPTRUS standard design resulted from the y displacement, and the compressive stress in bar 3. The squared-error for the OPTRUS local buckling constrained design resulted from the compressive stresses in bars 2 and 3. OPTRUS required 0.335 CPU seconds for the solution of the $s=0$ problem, and 0.378 CPU seconds for the solution of the $s=150$ problem.

Ten-Bar Truss

The structural geometry, constraints, and mechanical loads are as defined in Ref. 6. The minimization procedures initiated with all $A_i=193.56$ cm^2. A comparison of minimum-mass designs is presented in Table 3. The squared-error for the OPTRUS standard design resulted from the vertical displacement at nodes 1 and 2. The squared-error for

Table 3 Minimum-mass design of ten-bar truss

	Standard design, $s=0$[a]			OPTRUS design	
	Ref. 6	Ref. 7	Ref. 8	$s=0$	$s=150$
Areas, cm^2					
A_1	196.786	197.883	172.772	186.451	149.709
A_2	0.645	0.645	0.645	0.699	25.698
A_3	150.267	153.300	165.081	153.933	299.232
A_4	99.541	94.135	95.541	97.897	143.309
A_5	0.645	0.645	2.213	0.645	0.645
A_6	1.355	0.645	0.645	0.645	24.234
A_7	49.351	55.345	51.848	58.489	62.321
A_8	135.362	135.944	132.169	128.663	206.092
A_9	140.770	135.234	149.035	141.238	81.029
A_{10}	0.645	0.645	0.645	0.847	16.7091
Mass, kg	2304.29	2302.86	2300.11	2292.73	2937.51
Structural analyses	15	13	20	19	37
Optimization analyses	14	13	20	13	23
Final squared error	0	0	6.5×10^{-4}	1.4×10^{-4}	7.9×10^{-8}

[a] Slenderness ratio.

Table 4 Results of application of OPTRUS to twenty-five-bar truss, $s=150$[a]

Areas, cm^2	
A_1	0.06452
A_2, A_3, A_4, A_5	3.77612
A_6, A_7, A_8, A_9	24.4298
A_{10}, A_{11}	0.06743
A_{12}, A_{13}	13.4005
$A_{14}, A_{15}, A_{16}, A_{17}$	6.13147
$A_{18}, A_{19}, A_{20}, A_{21}$	0.411755
$A_{22}, A_{23}, A_{24}, A_{25}$	29.2005
Mass, kg	244.393
Structural analyses	28
Optimization analyses	18
Final squared error	1.5×10^{-9}

[a] Slenderness ratio.

Table 5 Additional details of minimum-mass designs with $s=150$[a]

Design variable	Minimum acceptable radius of gyration, cm 3-bar	4-bar	10-bar	25-bar
1	2.39474	2.79355	6.09601	1.27000
2	1.69334	2.27185	6.09601	2.20987
3		3.15123	6.09601	1.80848
4		3.56614	6.09601	1.27000
5			6.09601	1.27000
6			6.09601	3.06734
7			8.62106	3.06734
8			8.62106	2.25999
9			8.62106	
10			8.62106	
Fundamental natural frequency ω, rad/s	630.08	475.37	72.822	85.473
Buckling load factor, ϕ	1.15162×10^4	2.09532×10^2	5.46854×10^1	2.79573

[a] Slenderness ratio.

the OPTRUS local buckling constrained design resulted from the vertical displacement at node 2. OPTRUS required 0.346 CPU seconds for the solution of the $s=0$ problem, and 0.382 CPU seconds for the solution of the $s=150$ problem.

Twenty-five-Bar Truss

The structural geometry and constraints are as defined in Ref. 9. The mechanical loads are as defined as load case one in Ref. 9. The OPTRUS procedure initiated with all $A_i=12.904$ cm^2 with the results presented in Table 4. The squared-error resulted from the compressive stress in bar 25. The solution required 0.388 CPU seconds.

Additional details of the buckling constrained minimum-mass designs are presented in Table 5.

Table 6 Results of application of OPTRUS2 to three-bar truss

Areas A_1 and A_3, cm^2	5.72922
Area A_2, cm^2	1.77704
Lumped mass, kg	0.28586
Mass, kg	12.9272
Structural analyses	14
Optimization analyses	11
Fundamental natural frequency ω, rad/s—	
Equality constraint	600.000
Obtained	600.000
Final squared error	1.8×10^{-10}

OPTRUS2 Applications

The OPTRUS2 routine seeks the solution of a problem in which unequal nodal temperatures set up internal forces which are a linear function of the element areas. If an element area is increased to lower the stress resulting from a mechanical load, the thermal load will increase. It is recognized that it is possible to encounter structures for which no resizing of the element areas will result in an acceptable design. A second difficulty is in attempting to satisfy the highly nonlinear equality constraint on the free vibration natural frequency. The magnitudes of the limiting nodal displacements were 0.762 and 1.270 cm for the three- and four-bar structures, respectively. Otherwise the OPTRUS2 routine sought the minimum-mass designs for each of the four structures under the same displacement and stress constraints as the local buckling constrained designs. For each structure, the fundamental frequency was sought to equal a number arbitrarily smaller than the TRUSAN determined fundamental frequencies of the local buckling constrained designs (Table 5). An additional design variable was introduced for each structure. Physically, this variable represented a lumped mass acting at each node. For each structure, the OPTRUS2 routine was initiated with the element areas of the final local buckling constrained designs, and the lumped mass value set to 14.594 kg.

Table 7 Results of application of OPTRUS2 to four-bar truss

Areas, cm^2	
A_1	17.1848
A_2	84.4786
A_3	79.3854
A_4	0.0
Lumped mass, kg	6.00526
Mass, kg	210.225
Structural analyses	7
Optimization analyses	6
Fundamental natural frequency ω, rad/s—	
Equality constraint	400.000
Obtained	400.005
Final squared error	7.3×10^{-8}

Table 8 Results of application of OPTRUS2 to ten-bar truss

Areas, cm^2	
A_1	151.499
A_2	10.3271
A_3	296.303
A_4	149.042
A_5	0.6452
A_6	15.2236
A_7	57.7875
A_8	212.518
A_9	108.576
A_{10}	9.25514
Lumped mass, kg	5.82957
Mass, kg	2989.333
Structural analyses	20
Optimization analyses	13
Fundamental natural frequency ω, rad/s—	
Equality constraint	70.0000
Obtained	70.0007
Final squared error	3.1×10^{-6}

Table 9 Results of application of OPTRUS2 to twenty-five-bar truss

Areas, cm^2	
A_1	0.06452
A_2, A_3, A_4, A_5	3.35920
A_6, A_7, A_8, A_9	24.7211
A_{10}, A_{11}	0.06452
A_{12}, A_{13}	19.9025
$A_{14}, A_{15}, A_{16}, A_{17}$	5.71401
$A_{18}, A_{19}, A_{20}, A_{21}$	0.456798
$A_{22}, A_{23}, A_{24}, A_{25}$	29.1292
Lumped mass, kg	1.4124×10^{-3}
Mass, kg	248.425
Structural analyses	17
Optimization analyses	12
Fundamental natural frequency ω, rad/s—	
Equality constraint	80.0000
Obtained	80.2268
Final squared error	4.9×10^{-5}

The thermal loads considered were as follows.

Three-Bar Truss. The temperatures of nodes 1-4 were 338.5, 310.8, 283.0, and 255.2 K.

Four-Bar Truss. Node 1 was set to a temperature of 255.2 K, nodes 2 and 3 to 366.3 K, and nodes 4 and 5 to 310.8 K.

Ten-Bar Truss. Nodes 1 and 2 were set to 302.4 K, nodes 3 and 4 to 283.0 K, and nodes 5 and 6 to 263.6 K.

Twenty-five-Bar Truss. Nodes 1 and 2 were set to 302.4 K, nodes 3-6 to 283.0 K, and nodes 7-10 to 263.6 K.

The performance of OPTRUS2 is summarized in Tables 6-9. The three bar truss solution required 0.227 CPU seconds. The squared-error resulted from the tensile stress in bar 2, and the compressive stress in bar 3. The four-bar truss solution required 0.263 CPU seconds. The squared-error resulted from the tensile stress in bar 1, the compressive stress in bars 2 and 3, and the natural frequency. The ten-bar truss solution required 1.136 CPU seconds. The squared-error resulted from the vertical displacement at node 2 and the natural frequency. The twenty-five bar truss solution required 7.281 CPU seconds. The squared-error resulted from the compressive stress in bars 2 and 25, and the natural frequency.

Consistent with requirements to lower fundamental frequency, the OPTRUS2 solutions are all more massive than the respective OPTRUS local buckling constrained solutions. Only in the OPTRUS2 solution for the twenty-five-bar truss was the mass increase obtained without significant lumped mass. The OPTRUS2 four-bar truss solution shows all the necessary mass change from the lumped mass. The OPTRUS2 solutions show decreases in individual element areas when compared to the respective OPTRUS local buckling constrained solutions. These decreases in individual element areas are an indication of the nonlinearities introduced with the thermal loads. In the absence of lumped masses, it should be recognized that it is possible to encounter structures for which no resizing of the element areas will result in an acceptable design.

Concluding Remarks

A penalty function procedure was presented for determining the minimum-mass design of truss structures under static mechanical and thermal loads with constraints on the design variables, nodal displacements, member stresses, local buckling, and fundamental natural frequency. The procedure employs the square roots of both element areas, and lumped masses, as design variables. Thus, the first and second derivatives of the quadratic objective function are readily available. Also, the design variables may go to zero without complication. The procedure employs an exterior penalty function to transform the constrained objective function into an unconstrained index of performance which is minimized by Gauss' method. Gauss' method recasts the minimization problem to one of solving simultaneous linear equations with the variation of the parameters as the unknowns. Equality and inequality constraints may be considered. Only the first derivatives of the violated constraints are required for the solution. The procedure is amenable to several techniques for computational efficiency including the use of approximate structural analyses, storing only necessary components of mass and stiffness matrices as vectors, and avoiding unnecessary stiffness matrix decomposition. The procedure is simple to implement. Solutions can be sought beginning with an infeasible design point.

The natural frequency analysis procedure used a four-degree-of-freedom axial-force bar element providing a substantial improvement in accuracy relative to that available when using the standard two-degree-of-freedom axial-force bar element. The accuracy improvement was achieved while retaining frequency independent stiffness and mass matrices.

The procedure was applied to some standard test problems to obtain minimum-mass designs of truss structures under static mechanical loads with constraints on design variables, nodal displacements, and member stresses. The procedure showed very good results relative to previously published work. The standard test problems were modified to introduce local buckling constraints by changing the allowable compressive stress in each member to reflect a selected allowable slenderness ratio. The procedure obtained four new designs. To demonstrate the full capabilities of the procedure, each standard test problem was further modified by the introduction of thermal loads, and an equality constraint on the fundamental natural frequency.

The procedure is offered as another candidate for application in minimum-mass truss structure design. Along with some previously published approaches, the basic procedure can be modified to attempt the minimum-mass design of more general structures. As presented, the procedure has demonstrated potential for application in the design of large space structures.

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